

# Remarks on the mixed Ramond -Ramond, open string scattering amplitudes of BPS, non-BPS and brane-anti brane

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## Abstract

From the world-sheet point of view we compute three, four and five point BPS and non-BPS scattering amplitudes of type IIA and IIB superstring theory. All these the mixed S-matrix elements including a closed string Ramond-Ramond (RR) in the bulk and a scalar/gauge or tachyons with their all different pictures ( including RR in asymmetric and symmetric pictures) have been carried out. We have also shown that in asymmetric pictures various equations must be kept fixed. More importantly, by direct calculations on upper half plane, it is realized that some of the equations (that must be true) for BPS branes can not be necessarily applied to non-BPS amplitudes. We also derive the S-Matrix elements of  $\langle V_C^{-2} V_\phi^0 V_A^0 V_T^0 \rangle$  and clarify the fact that in the presence of the scalar field and RR, the terms carrying momentum of RR in transverse directions play important role in the entire form of the S-matrix and their presence is needed in order to have gauge invariance for the entire S-matrix elements of type IIA (IIB) superstring theory.

# 1 Introduction

In a very important paper [1], it has been extensively clarified that the sources for all different kinds of D-branes are Ramond- Ramond (RR) fields. It is worthwhile looking at some concrete references regarding RR fields [2, 3]. Besides them, RR fields play a very crucial effect in understanding the phenomenon of Dielectric branes which was first demonstrated by Myers in [4]. Having employed several RR couplings of [5], we could explore and interpret the  $N^3$  entropy of  $M5$  branes as well.

It is also known that if one wants to work with the effective actions of type IIA,IIB string theory, then one needs to deal with DBI and Chern-Simons effective actions which can be accordingly found in [6],[7],[8] and [9]<sup>1</sup>.

By making use of the scattering theory of D-branes in the world volume of BPS branes in type II string theory we have also explored various new Chern-Simons couplings including their all order  $\alpha'$  corrections to the low energy effective actions of D-branes. In fact it is in detail explained that for BPS (non-BPS) branes all the corrections to D-brane effective actions can just be derived by having the entire form of S-matrix and not by any other tools like T-duality transformation. The reason for this sharp conclusion is that by having S-matrix we are able to actually fix all the coefficients of the effective field theory couplings (and also their higher order  $\alpha'$  corrections) without any ambiguities ( see [11, 12] for BPS and [13] for non-BPS branes). In fact all the three different ways of obtaining the couplings in the effective field theory involving pull-back, Taylor expansions and new Wess-Zumino terms (the generalization to Myers action) have also been explained in [11].

It is emphasized in [14, 15, 16] that to get to the effective theory of non supersymmetric cases or non-BPS branes one has to integrate out all the massive modes and needs to effectively work out just with the massless strings such as scalar, gauge fields and also employ the real components of tachyon fields.

There are various motivations to perform scattering theory of all BPS and non-BPS branes , basically one of the main reasons to employ it, is to actually have the entire form of S-matrix elements which is a physical quantity and the other motivation would be to deal with its strong potential of gaining new terms including their corrections (of course without any ambiguity) to string theory effective actions.

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<sup>1</sup>Some of the new curvature corrections of type II have been recently obtained in [10].

Having set this formalism, one may end up obtaining several new couplings for all BPS branes in RNS formalism [17, 18] and eventually one could investigate to get to a proposal for the corrections to some of the couplings [19]. In particular it is shown that some of these couplings must be employed to be able to work with some of the applications of either M-theory [20] or gauge-gravity duality [21].

One might look for various applications in the world volume of non-BPS branes as some of them were comprehensively pointed out in [13], but for the concreteness we highlight some of them again.

As an instance in [22] it has been illustrated that as long as the effective field theory description holds, in the large volume case (despite having the non supersymmetric cases), the Ads minima are indeed stable vacua. Not only the phenomena of the production of branes are investigated in [23, 24] but also Inflation in the language of D-brane and anti D-brane systems in string theory was revealed [25, 26, 27, 28]. It is also worth mentioning that tachyons of type IIA(IIB) superstring theory (with odd-parity) have been taken into account to make various remarks on some holographic QCD models [29, 30] as well.

In [31] various issues on the scattering amplitudes have been extensively discussed, however, one has to concern about some other issues on the mixed amplitudes involving a closed string Ramond-Ramond (RR) and some other open strings for which some of them have been empirically addressed in [19]. The content of this paper is beyond what has been appeared in those references. Indeed we would like to understand in the presence of symmetric and asymmetric picture of RR, a scalar field and some other open strings what happens to the terms that carrying momentum of RR in transverse directions, arguing that the S-matrices that satisfy Ward identity and do involve all the contact interactions are definitely the correct S-matrices.

The paper organized as follows. In the next section, first we try to come up with the entire details of a three point <sup>2</sup> scattering computations of a closed string RR in asymmetric picture ( $C^{-2}$  picture, in terms of its potential not its field strength) and a scalar field. Then we carry out the same S-matrix in symmetric picture of RR ( $C^{-1}$  picture, in terms of RR's field strength) and scalar picture in  $(-1)$  picture. Finally we compare both S-matrixes and

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<sup>2</sup>three point function from the world sheet point of view and two point function from the space time prospective

make remarks on a Bianchi identity that must be true to get to picture independence result. For the completeness we talk about  $CT, CA$  amplitudes as well.

The four point correlation function of  $\langle T^{(0)}T^{(0)}C^{(-2)} \rangle$  in the world volume of D-brane anti D-brane system has also been carried out to show that the result is the same as  $\langle T^{(-1)}T^{(0)}C^{(-1)} \rangle$  [32]. This clearly confirms that there is no any issue of picture dependent of the mixed closed string RR and strings that move on the world volume space such as gauge fields or tachyons (but not scalar field).<sup>3</sup>

Hence due to momentum conservation along the world volume of branes and as long as we are dealing with world volume gauge fields and or tachyons in the presence of RR, there is no any issue about choosing the picture of RR (in symmetric or asymmetric )<sup>4</sup>.

However, because there is a non-zero correlation function between RR and scalar in zero picture and also due to the fact that the terms carrying momentum of RR in transverse directions  $(p^i, p^j)$  can not be derived by any duality transformations [11, 13, 33], one has to be concerned about various issues. In the following we release various subtleties that have to be considered for the mixed amplitudes involving a closed string RR, a scalar and some arbitrary numbers of gauges or tachyons in the world volume of BPS, brane/anti brane and non-BPS systems.

We then start calculating all the four point non-BPS functions of type II in their all different pictures  $\langle T^{(-1)}\phi^{(0)}C^{(-1)} \rangle, \langle T^{(0)}\phi^{(-1)}C^{(-1)} \rangle, \langle T^{(0)}\phi^{(0)}C^{(-2)} \rangle$ . In the case of  $\langle T^{(-1)}\phi^{(0)}C^{(-1)} \rangle$  we see the term that carries momentum of RR in transverse direction disappears after applying a Bianchi identity equation, however, we claim that one has to be careful about these transverse  $(p^i, p^j)$  terms in higher point functions as their presence plays crucial role in the gauge invariance of the higher point mixed amplitudes. There is a non-zero correlation function between RR and the first part of the vertex operator of scalar field in zero picture and therefore one needs to think about those terms that carry momentum of RR in transverse direction  $(p^i, p^j)$  as they can not be derived by any duality transformation [11]. Indeed by direct computations of scattering amplitudes of BPS branes, we observe that several Bianchi identity must hold for BPS cases, whereas

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<sup>3</sup> Because the polarization of scalar field is in the bulk and there is a non zero correlator between RR and the first term of scalar vertex operator in zero picture as  $\langle e^{ip \cdot x(z)} \partial_i x^i(x_1) \rangle$  is non zero. Basically one needs to concern about the terms that carry momentum of RR in transverse directions ( $p^i$ 's terms).

<sup>4</sup>In fact in order to get to the final answer for these S-matrices as fast as it is possible, one could put RR and a gauge field in  $(-1)$  picture and the other tachyons/gauges in zero picture.

we show that these equations can not be necessarily true for non-BPS branes (say for  $\langle T^{(0)}T^{(0)}T^{(0)}C^{(-2)} \rangle$ ), otherwise the whole non supersymmetric S-matrix vanishes. For the completeness we perform  $\langle \phi^{(0)}A^{(0)}C^{(-2)} \rangle$ ,  $\langle \phi^{(-1)}A^{(0)}C^{(-1)} \rangle$  and  $\langle \phi^{(0)}A^{(-1)}C^{(-1)} \rangle$  as well.

We also would like to go over to some of the mixed RR, scalar and tachyon five point functions of either brane/ anti brane or non-BPS branes to see what happens, if we carry them out in both symmetric and asymmetric picture of RR accordingly. In fact for a scattering amplitude of brane anti brane system (including scalar, tachyons and RR), we have three different choices.<sup>5</sup> Indeed for these particular amplitudes there is no Ward identity and a priori one does not know which specific picture gives us the correct S-matrix, where we claim and establish the fact that already at the level of S-matrix for brane / anti brane, one needs to know some generalised Bianchi identities to be able to produce all the effective field theory couplings.

Most importantly, we show that the terms that carry momentum of RR in transverse directions  $(p^i, p^j)$ , that are singular of  $\langle \phi^{(0)}T^{(-1)}T^{(0)}C^{(-1)} \rangle$  and  $\langle \phi^{(0)}T^{(0)}T^{(0)}C^{(-2)} \rangle$  amplitudes remain after taking integrations properly on the upper half plane, while these terms are absent in  $\langle \phi^{(-1)}T^{(0)}T^{(0)}C^{(-1)} \rangle$ . Hence in order to remove all the apparent singularities of brane/anti brane, we introduce new Bianchi identities at the level of world-sheet.

We then move on to obtain  $\langle T^{(0)}T^{(0)}T^{(0)}C^{(-2)} \rangle$  S-matrix and determine the fact that if we apply some of the Bianchi identities of BPS branes to this non-BPS amplitude then the whole S-matrix disappears so this clearly confirms that those Bianchi equations of BPS branes must not be true for non-BPS amplitudes. The reason is that, there are non zero field theory couplings in the world volume of non-BPS branes that have to be produced by non-zero S-matrix of an RR and three tachyons. We will also mention several subtleties that might have potentially something to do with some of the issues on the perturbative string theory that are released and have been pointed out in a series of papers appeared by Witten [34].

The five point correlations of  $\langle V_C^{-1}V_\phi^{-1}V_T^0V_A^0 \rangle$  have been computed in [35]. In order to get to know what happens to the gauge invariance of the amplitudes we come over to the same amplitude but with different picture of scalar as  $\langle V_C^{-1}V_\phi^0V_A^{-1}V_T^0 \rangle$ .

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<sup>5</sup>  $\langle \phi^{(-1)}T^{(0)}T^{(0)}C^{(-1)} \rangle$ ,  $\langle \phi^{(0)}T^{(-1)}T^{(0)}C^{(-1)} \rangle$  and  $\langle \phi^{(0)}T^{(0)}T^{(0)}C^{(-2)} \rangle$ .

Given the fact that the vertex operator of a scalar field in zero picture carries two different terms and in particular its first part has non zero correlation function with closed string RR, we see that here the terms carrying momentum of RR (all  $p.\xi$  terms) survive after taking integration on upper half plane. In fact due to all non vanishing  $p.\xi$  terms the final form of the S-matrix does not satisfy Ward identity associated to gauge field unless we introduce new Bianchi identities. Since we can not give up gauge invariance of the S-matrix, we need to come up with some ideas. That is why we look at the same S-matrix in an asymmetric picture of RR  $\langle V_C^{-2}V_\phi^0V_T^0V_A^0 \rangle$  where in this picture upon considering the known Bianchi identities not only will we observe that the amplitude respects the Ward identity associated to the gauge field, but also we are able to get to the whole contact interactions of the related S-matrix.

We may wonder why we can not see the term that carries momentum of RR in transverse direction in four point functions, say  $\langle V_C^{-1}V_\phi^0V_A^{-1} \rangle$ . The answer is that in these functions we do have that particular term, however, after gauge fixing the integral should be taken on the whole space time (from  $-\infty$  to  $\infty$ ) and since the integrand (including  $p^i$  term) is odd, the final result naturally is zero. But this does not happen for five point functions any more, basically, for five and higher point functions after fixing  $SL(2,R)$  invariance we need to take the integrals on the position of closed string and the remaining terms involving  $(p^i, p^j)$  terms are not vanished. Even these terms might not satisfy Ward identity. The resolution to this, is to either introduce some new Bianchi identities or calculate all of the mixed S-matrixes in symmetric/ asymmetric pictures. Let us get to the technical parts by computing three point functions.

## 2 The $\phi^{(0)} - C^{(-2)}$ amplitude

In this section we are going to derive the full S-matrix elements of one scalar field and one closed string Ramond-Ramond (RR) in type IIA (IIB) String theory, where for some reasons we would like to keep RR in its asymmetric picture. That is, we consider its vertex operator in terms of its potential (not its field strength) so we deal with RR in  $C^{-3/2, -1/2}$  picture.

Motivation for doing this computation in different pictures is that, there is no Ward identity for the mixed RR and a scalar field and one might wonder what happens in higher point functions of all mixed amplitudes including RR, scalar field and tachyons (but not

gauge field), or one might ask which particular picture is going to give us the complete S-matrix elements including all the infinite contact interactions of string theory.

Note that our notations are such that  $\mu, \nu, ..$  run over the whole space-time,  $a, b, c, ..$  and  $i, j, k, ..$  are world volume and transverse directions accordingly.

Thus, this four point function from world-sheet point of view ( three point function, from the space-time view) of one scalar and an asymmetric RR closed string is given by its following correlation function

$$\mathcal{A}^{C^{(-2)\phi^{(0)}}} \sim \int dx dz d\bar{z} \langle V_{RR}^{(-2)}(z, \bar{z}) V_\phi^{(0)}(x) \rangle \quad (1)$$

Note that the vertex of RR in asymmetric picture has been first proposed by [36] so the vertices can be read off as follows

$$\begin{aligned} V_\phi^{(0)}(x) &= \xi_{1i}(\partial^i X(x) + i\alpha' k \cdot \psi \psi^i(x)) e^{\alpha' i k \cdot X(x)} \\ V_{RR}^{(-2)}(z, \bar{z}) &= (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} e^{-3\phi(z)/2} S_\alpha(z) e^{i\frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i\frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \end{aligned}$$

We are considering the disk level amplitude so the RR has to be put in the middle of disk while the scalar field needs to be replaced just in its boundary. On-shell conditions for scalar and RR are <sup>6</sup>

$$k^2 = p^2 = 0, \quad k \cdot \xi_1 = 0$$

For simplicity it is really useful to just make use of World-sheet's holomorphic elements, which means that we employ some valuable change of variables as follows

$$\tilde{X}^\mu(\bar{z}) \rightarrow D_\nu^\mu X^\nu(\bar{z}), \quad \tilde{\psi}^\mu(\bar{z}) \rightarrow D_\nu^\mu \psi^\nu(\bar{z}), \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z}), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \rightarrow M_\alpha^\beta S_\beta(\bar{z}),$$

Some definitions might be important to highlight as well . <sup>7</sup>

Applying those vertex operators and Wick theorem, our desired S-matrix can be written down by

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<sup>6</sup> The definition of projector and the field strength of closed string is

$$P_- = \frac{1}{2}(1 - \gamma^{11}), \quad \mathbb{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n}$$

where for type IIA (type IIB)  $n = 2, 4, a_n = i$  ( $n = 1, 3, 5, a_n = 1$ ) with  $(P_- \mathbb{H}_{(n)})^{\alpha\beta} = C^{\alpha\delta} (P_- \mathbb{H}_{(n)})_{\delta}{}^{\beta}$  notation for spinor .

<sup>7</sup>

$$D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix}, \quad \text{and} \quad M_p = \begin{cases} \frac{\pm i}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \epsilon_{i_1 \dots i_{p+1}} & \text{for p even} \\ \frac{\pm 1}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \gamma_{11} \epsilon_{i_1 \dots i_{p+1}} & \text{for p odd} \end{cases}$$

$$\int dx_1 dx_4 dx_5 (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} (x_{45})^{-3/4} \xi_{1i} \left( I_1 + (2ik_{1a}) I_2 \right) I$$

so that  $x_4 = z = x + iy$ ,  $x_5 = \bar{z} = x - iy$  and

$$I = |x_{14} x_{15}|^{\frac{\alpha'^2}{2} k_1 \cdot p} |x_{45}|^{\frac{\alpha'^2}{4} p \cdot D \cdot p}, \quad (3)$$

with

$$I_1 = ip^i (x_{54})^{-5/4} C_{\alpha\beta}^{-1} \frac{x_{54}}{x_{14} x_{15}} \quad (4)$$

It is also important to talk about the following correlator which can be obtained by generalising the Wick-like rule [37, 38]

$$I_2 = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a \psi^i(x_1) : \rangle = 2^{-1} (x_{14} x_{15})^{-1} (x_{45})^{-1/4} (\Gamma^{ia} C^{-1})_{\alpha\beta}$$

If we would replace the above correlators inside the amplitude then we could see that our S-matrix does respect the  $SL(2, R)$  invariance. We gauge fix it as  $(x_1, z, \bar{z}) = (\infty, i, -i)$  so that the final result of our S-matrix in this certain picture is

$$\mathcal{A}^{\phi^0, C^{-2}} = \left[ -ip^i \text{Tr} (P_- \mathcal{C}_{(n-1)} M_p) + ik_{1a} \text{Tr} (P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ia}) \right] \xi_{1i}. \quad (5)$$

As it can be seen from the above S-matrix, it seems to have two different terms in our amplitude while in below we show that some crucial subtleties are needed. Note that after the derivation of the S-matrix, one could start writing all its field theory couplings to be compared with the amplitude while before doing so, we claim that one has to know the correct form of the S-matrix. Hence let us carry out this amplitude in the other picture  $\langle C^{(-1)} \phi^{(-1)} \rangle$  in the next section and back to the subtlety associated to (5) afterwards.

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where now the propagators for all world-sheet fields are

$$\begin{aligned} \langle X^\mu(z) X^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z-w), \\ \langle \psi^\mu(z) \psi^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} (z-w)^{-1}, \\ \langle \phi(z) \phi(w) \rangle &= -\log(z-w). \end{aligned} \quad (2)$$



It is also worth deriving the S-matrix of one RR and a gauge field in the asymmetric picture of RR.<sup>8</sup>

### 3 The $C^{-1} - \phi^{-1}$ amplitude

The three point function from the world-sheet prospective ( two point function from the space time point of view ) of one closed string RR and a real tachyon of type II super string in their different pictures has been done.<sup>9</sup>

This three point function from the world-sheet prospective with both RR and scalar field in  $(-1)$  picture is given by

$$\mathcal{A}^{\phi, RR} \sim \int dx d^2 z \langle V_{\phi}^{(-1)}(x) V_{RR}^{(-1)}(z, \bar{z}) \rangle \quad (6)$$

The vertices are written down so that RR is considered in terms of its field strength in symmetric picture. They are presented as follows

$$\begin{aligned} V_{\phi}^{(-1)}(x) &= e^{-\phi(x)} \xi_i \psi^i(x) e^{2iqX(x)} \\ V_{RR}^{(-1)}(z, \bar{z}) &= (P_- \not{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_{\alpha}(z) e^{i\frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_{\beta}(\bar{z}) e^{i\frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \end{aligned}$$

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<sup>8</sup> Let us just mention the final answer

$$\mathcal{A}^{A^0, C^{-2}} = \xi_{1a} \left[ -ip^a \text{Tr}(P_- \not{C}'_{(n-1)} M_p) + ik_{1b} \text{Tr}(P_- \not{C}'_{(n-1)} M_p \Gamma^{ab}) \right]$$

Note that the first term in above S-matrix has definitely no contribution to the amplitude because if we apply momentum conservation along the world volume of brane  $(k_1 + p)^a = 0$  and on-shell condition for the gauge field gives rise the first part of the S-matrix to be vanished. Indeed the second part of the S-matrix can be produced by  $2\pi\alpha' \int C_{p-1} \wedge F$  coupling as the three point function of a symmetric closed string RR and one gauge field in  $(-1)$  picture was given by

$$\begin{aligned} V_A^{(-1)}(x) &= e^{-\phi(x)} \xi_a \psi^a(x) e^{2iqX(x)} \\ \mathcal{A}^{C^{-1} A^{-1}} &\sim 2^{-1/2} \xi_{1a} \text{Tr}(P_- \not{H}_{(n)} M_p \gamma^a) \end{aligned}$$

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$$\mathcal{A}^{C^{-1} T^{-1}} \sim -2i \text{Tr}(P_- \not{H}_{(n)} M_p)$$

and also

$$\mathcal{A}^{C^{-2} T^0} \sim 2^{1/2} (2ik_{1a}) \text{Tr}(P_- \not{C}'_{(n-1)} M_p \gamma^a)$$

now if we apply momentum conservation along the world volume of brane  $(k_1 + p)^a = 0$ , extract the trace and use  $p \not{C}' = \not{H}$  (up to normalisation constant) we get the same S-matrix in both pictures, where this S-Matrix can be generated with  $2i\pi\alpha' \beta' \mu'_p \int C_p \wedge DT$  coupling in field theory.

Obviously all the previous definitions of the first section for projector, holomorphic components and the other field contents have been held here as well. Once more we substitute the defined vertex operators into (6) and the amplitude reduces to

$$\int dx_1 dx_4 dx_5 (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} (x_{14} x_{15})^{-1/2} (x_{45})^{-1/4} \xi_{1i} I \\ \times \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^i(x_1) : \rangle$$

where the result for the following correlation function is needed, that is ,

$$\langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^i(x_1) : \rangle = 2^{-1/2} (x_{14} x_{15})^{-1/2} (x_{45})^{-3/4} (\gamma^i C^{-1})_{\alpha\beta}$$

The  $SL(2, R)$  invariance of the S-matrix can be readily checked and we did gauge fixing as  $(\infty, i, -i)^{10}$ . The final result of the S-matrix of one scalar and one RR closed string in this symmetric picture is

$$\mathcal{A}^{C^{-1}\phi^{-1}} = 2^{-1/2} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^i) \xi_{1i} . \quad (7)$$

One should pay particular attention to the the conservation of momentum along the world volume of brane as  $k_1^a + p^a = 0$ . Let us first reproduce the field theory of above S-Matrix. The amplitude might be normalised by a coefficient of  $(2^{1/2} \pi \mu_p / 8)$  such that  $\mu_p$  is Ramond-Ramond charge of brane. The trace is done as follows<sup>11</sup>

$$\text{Tr} \left( \mathbb{H}_{(n)} M_p \gamma^i \right) \delta_{p+2,n} = \pm \frac{32}{(p+2)!} \epsilon^{a_0 \dots a_p} H_{ia_0 \dots a_p} \delta_{p+2,n}$$

Eventually this S-matrix (7) can be precisely produced with the following field theory coupling

$$\mu_p (2\pi\alpha') \int_{\Sigma_{p+1}} \left( \text{Tr} (\partial_i C_{p+1} \phi^i) \right) \quad (8)$$

where we have used the so called Taylor expansion of a scalar field, meanwhile in the other picture , the S-matrix was found to be

$$\mathcal{A}^{\phi^0, C^{-2}} = \left[ -ip^i \text{Tr} (P_- \mathcal{Q}'_{(n-1)} M_p) + ik_{1a} \text{Tr} (P_- \mathcal{Q}'_{(n-1)} M_p \Gamma^{ia}) \right] \xi_{1i}$$

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<sup>10</sup> we set  $\alpha' = 2$

<sup>11</sup>The trace with  $\gamma^{11}$  can be shown that all the above results kept held even for the following case

$$p > 3, H_n = *H_{10-n}, n \geq 5$$

Let us compare (5) with (7). We know that  $p^i C = H^i$  so up to normalisation constant the first term of (5) does exactly produce the same S-matrix of (7), therefore we claim that the second term of (5) has no contribution to the S-matrix of one scalar and one RR at all. Hence the prescription for removing or getting rid of the second term of (5) is as follows.

We first apply the momentum conservation along the world volume of brane to the second term ( $k_1^a + p^a = 0$ ) and then extract its trace as follows:

$$\text{Tr} \left( \mathcal{C}_{(n-1)} M_p \Gamma^{ia} \right) \delta_{p+2,n} = \pm \frac{32}{(p+1)!} \epsilon^{a_0 \dots a_{p-1} a} C_{ia_0 \dots a_{p-1}} \delta_{p+2,n}$$

and more importantly in order to get to the same S-matrix as  $\mathcal{A}^{\phi^{-1}, C^{-1}}$ , we understand that the following Bianchi identity must hold for BPS branes

$$p^a \epsilon^{a_0 \dots a_{p-1} a} = 0 \quad (9)$$

However, this is not the full story, as we will see in the next sections for the higher point functions of string amplitudes, one has to generalise all the Bianchi identities<sup>12</sup>. Let us now turn to some of the four point functions and obtain new Bianchi identities.

## 4 The $T^{-1} - \phi^0 - C^{-1}$ amplitude

The four point function from the world-sheet prospective ( three point function from the space time point of view ) of one closed string RR and two real tachyons of type II super string with their all different pictures can be done.<sup>13</sup>

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<sup>12</sup>In particular in order not to miss any contact interactions, one might need to look at the other pictures of the higher point functions of string theory amplitudes as well.

<sup>13</sup> Indeed after performing the gauge fixing as  $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$  with  $u = -\frac{\alpha'}{2}(k_1 + k_2)^2$  the S-matrix is

$$\begin{aligned} V_T^{(-1)}(y) &= e^{-\phi(y)} e^{2ik \cdot X(y)} \otimes \sigma_2 \\ \mathcal{A}^{C^{-1} T^{-1} T^0} &\sim 2^{3/2} \pi \frac{\Gamma[-2u]}{\Gamma[1/2 - u]^2} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^a) k_{2a} \end{aligned} \quad (10)$$

where one can use momentum conservation along the brane  $-k_1^a - p^a = k_2^a$  and apply the following Bianchi identity  $p_a \epsilon^{a_0 \dots a_{p-1} a} = 0$ , to show that the S-matrix is antisymmetric with respect to interchanging the two tachyons, whereas this S-matrix in asymmetric picture is

$$\mathcal{A}^{C^{-2} T^0 T^0} \sim 4k_{1a} k_{2b} \int_{-\infty}^{\infty} dx (2x)^{-2u-1} \left( (1+x^2) \right)^{2u} \left[ \text{Tr} (P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ba}) - 2\eta^{ab} \frac{1-x^2}{4xi} \text{Tr} (P_- \mathcal{C}_{(n-1)} M_p) \right],$$

The four point function of one tachyon, a scalar and one RR from the world-sheet point of view has been performed in detail in [35]. Indeed both the S-matrix and its field theory part in the following picture  $T^0 - \phi^{-1} - C^{-1}$  have already been computed. Truly, after carrying out the gauge fixing as  $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$  the S-matrix is given by<sup>14</sup>

$$4k_{1a}\xi_i \int_{-\infty}^{\infty} dx (2x)^{-2u-1/2} \left( (1+x^2) \right)^{-1/2+2u} \left( \text{Tr} (P_- \mathbb{H}_{(n)} M_p \Gamma^{ia}) \right),$$

Thus the result of this non super symmetric amplitude is read off as

$$\mathcal{A}^{T^0, \phi^{-1}, C^{-1}} = (\pi\beta'\mu'_p) 2\sqrt{\pi} \frac{\Gamma[-u+1/4]}{\Gamma[3/4-u]} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \Gamma^{ai}) k_{1a} \xi_i \text{Tr} (\lambda_1 \lambda_2) . \quad (11)$$

where  $(\pi\beta'\mu'_p/2)$ ,  $\beta'$  and  $\mu'_p$  are defined as normalisation constant, WZ and the RR charge of brane. In effective field theory it was shown that, this S-matrix can be precisely reproduced by the following coupling of type II string theory

$$2i\beta'\mu'_p (2\pi\alpha')^2 \int_{\Sigma_{p+1}} \left( \text{Tr} (\partial_i C_p \wedge DT \phi^i) \right), \quad (12)$$

and all its infinite corrections were derived in [35].<sup>15</sup> Let us carry it out in the other picture so if we use the above vertex operators and perform all the correlators with same techniques that have been explained in the previous section, then one explores the final form of the S-matrix in this picture as

$$\begin{aligned} \mathcal{A}^{C^{-1}\phi^0 T^{-1}} &\sim 4\xi_{1i} \int_{-\infty}^{\infty} dx (2x)^{-2u-1/2} \left( (1+x^2) \right)^{2u-1/2} \\ &\times \left[ k_{1a} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \Gamma^{ia}) - p^i \text{Tr} (P_- \mathbb{H}_{(n)} M_p) \right], \end{aligned} \quad (14)$$

where evidently the second term has no contribution to the S-matrix, because the integration must be taken over the whole space-time and the integrand is odd function so the result for the second term is zero. Now if we apply momentum conservation to the first term of above S-matrix and more significantly in order to make sense of non-zero S-matrix in this asymmetric picture, we believe that the following equation

$$p_a \epsilon^{a_0 \dots a_{p-2} b a}$$

must be non-zero, otherwise the whole S-matrix vanishes.

Notice that all u-channel poles with an infinite higher derivative corrections to  $(2\pi\alpha')^2 \beta' \mu'_p \int C_{p-1} \wedge DT \wedge DT$  coupling can also be derived.

<sup>14</sup> with  $u = -\frac{\alpha'}{2}(k_1 + k_2)^2$  and  $(k_1^a + k_2^a + p^a = 0)$ .  
<sup>15</sup>

$$\frac{2\beta'\mu'_p}{p!} (2\pi\alpha')^2 \epsilon^{a_0 \dots a_p} \partial_i C_{a_0 \dots a_{p-1}} \wedge \text{Tr} \left( \sum_{m=-1}^{\infty} c_m (\alpha')^{m+1} D_{a_1} \dots D_{a_{m+1}} D_{a_p} T D^{a_1} \dots D^{a_{m+1}} \phi^i \right) \quad (13)$$

Now in order to get to the same S-matrix element for this four point world sheet amplitude as appeared in (11), one has to apply the momentum conservation  $(k_1 + k_2 + p)^a = 0$  and keep in mind the following Bianchi identity as well.

$$p^i \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p} + p^a \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}}^i = 0 \quad (15)$$

Finally let us calculate this S-matrix in asymmetric picture of RR and make some essential comments about this four point function.

Notice that there is a non zero coupling between two gauge fields and one RR in the world volume of BPS branes of type II string theory <sup>16</sup>

## 5 The $T^0 - \phi^0 - C^{-2}$ amplitude

The four point function of an asymmetric RR, a scalar and an open string tachyon can be investigated by the following correlation function

$$\mathcal{A}^{T^0 \phi^0 C^{-2}} \sim \int dx_1 dx_2 d^2 z \langle V_\phi^{(0)}(x_1) V_T^{(0)}(x_2) V_{RR}^{(-2)}(z, \bar{z}) \rangle \quad (16)$$

where the tachyon, scalar field and RR vertex operators are given as <sup>17</sup>

$$\begin{aligned} V_T^{(0)}(y) &= \alpha' i k_1 \cdot \psi(y) e^{\alpha' i k_1 \cdot X(y)} \lambda \otimes \sigma_1 \\ V_\phi^{(0)}(x) &= \xi_{1i} (\partial^i X(x) + i \alpha' k \cdot \psi \psi^i(x)) e^{\alpha' i k \cdot X(x)} \lambda \otimes I \\ V_{RR}^{(-2)}(z, \bar{z}) &= (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} e^{-3\phi(z)/2} S_\alpha(z) e^{i \frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i \frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \lambda \otimes \sigma_3 \sigma_1 \end{aligned}$$

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<sup>16</sup> In [39] it is shown that

$$\begin{aligned} \mathcal{A}^{C^{-1} A^{-1} A^0} \sim & 2^{-3/2} \xi_{1a} \xi_{2b} \int_{-\infty}^{\infty} dx (2x)^{-2u} \left( (1+x^2) \right)^{2u-1} \left\{ 4k_{2c} \text{Tr} (P_- \mathcal{H}_{(n)} M_p \Gamma^{bca}) + \left( \frac{1-x^2}{x} \right) \left[ k_{1b} \text{Tr} (P_- \mathcal{H}_{(n)} M_p \gamma^a) \right. \right. \\ & \left. \left. + k_{2c} \left( -\eta^{ac} \text{Tr} (P_- \mathcal{H}_{(n)} M_p \gamma^b) + \eta^{ab} \text{Tr} (P_- \mathcal{H}_{(n)} M_p \gamma^c) \right) \right] \right\} \end{aligned}$$

where obviously the last three terms of the S-matrix have no contribution to amplitude, because the integration must be taken over the whole space and the integrand is odd.

<sup>17</sup> To see the Chan-Paton factors see [35]

Applying Wick theorem ,the amplitude can be explored as follows

$$\int dx_1 dx_2 dx_4 dx_5 (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} (x_{45})^{-3/4} (4ik_{2a}) \xi_{1i} \left( I_3 + (2ik_{1c}) I_4 \right) I_5$$

with

$$\begin{aligned} I_5 &= |x_{12}|^{\alpha'_{2k_1.k_2}} |x_{14} x_{15}|^{\frac{\alpha'_{2k_1.p}}{2}} |x_{24} x_{25}|^{\frac{\alpha'_{2k_2.p}}{2}} |x_{45}|^{\frac{\alpha'_{4p.D.p}}{4}}, \\ I_3 &= ip^i 2^{-1/2} (x_{24} x_{25})^{-1/2} (x_{45})^{-3/4} (\gamma^a C^{-1})_{\alpha\beta} \frac{x_{54}}{x_{14} x_{15}} \end{aligned} \quad (17)$$

Now using Wick-like rule one gets to the following correlator

$$\begin{aligned} I_4 &= <: S_\alpha(x_4) : S_\beta(x_5) : \psi^c \psi^i(x_1) : \psi^a(x_2) : >= 2^{-3/2} (x_{24} x_{25})^{-1/2} (x_{14} x_{15})^{-1} (x_{45})^{1/4} \\ &\quad \left[ (\Gamma^{aic} C^{-1})_{\alpha\beta} + 2 \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} \eta^{ac} (\gamma^i C^{-1})_{\alpha\beta} \right] \end{aligned}$$

By applying the above correlators into this four point amplitude we can easily observe that the integrand or S-matrix is  $SL(2, R)$  invariant. We do the proper gauge fixing as  $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$ , taking  $u = -\frac{\alpha'}{2} (k_1 + k_2)^2$  we obtain the S-matrix as

$$\mathcal{A}^{T^0 \phi^0 C^{-2}} = \mathcal{A}_1 + \mathcal{A}_2$$

such that

$$\begin{aligned} \mathcal{A}_1^{T^0 \phi^0 C^{-2}} &= 2^{3/2} \xi_{1i} k_{2a} p^i \text{Tr} \left( \mathcal{C}_{(n-1)} M_p \gamma^a \right) \int_{-\infty}^{\infty} (2x)^{-2t-1/2} (x^2 + 1)^{2t-1/2} \\ &= 2^{3/2} \xi_{1i} 4k_{2a} p^i \text{Tr} \left( \mathcal{C}_{(n-1)} M_p \gamma^a \right) \sqrt{\pi} \frac{\Gamma[-u + 1/4]}{\Gamma[3/4 - u]} \end{aligned} \quad (18)$$

Now if we use the momentum conservation  $(k_1 + k_2 + p)^a = 0$  and the Bianchi identity  $p^a \epsilon^{a_0 \dots a_{p-1} a} = 0$ , then we come to the point that up to a coefficient of  $2^{3/2}$  this part of the S-matrix exactly produces (11). The second part of the S-matrix is found out to be

$$\begin{aligned} \mathcal{A}_2^{T^0 \phi^0 C^{-2}} &= 2^{3/2} \xi_{1i} k_{1c} k_{2a} \int_{-\infty}^{\infty} (2x)^{-2t-1/2} (x^2 + 1)^{2t-1/2} \left[ 2\eta^{ac} \text{Tr} \left( \mathcal{C}_{(n-1)} M_p \gamma^i \right) \frac{1 - x^2}{4xi} \right. \\ &\quad \left. + \text{Tr} \left( \mathcal{C}_{(n-1)} M_p \Gamma^{aic} \right) \right] \end{aligned} \quad (19)$$

where the first term in (19) is indeed zero because, the integration is taken over the whole space while the integrand is odd so naturally the answer for the first term of (19) is zero<sup>18</sup>

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$$2^{3/2} \xi_{1i} (-u - 1/4) \text{Tr} \left( \mathcal{C}_{(n-1)} M_p \gamma^i \right) \int_{-\infty}^{\infty} \left( \frac{(1+x^2)^2}{(4x^2)} \right)^{1/4+u} \frac{1-x^2}{(x^2+1)(4xi)} = 0 \quad (20)$$

However, as it is clear the result for the second term of (19) is non-zero, that is,

$$\mathcal{A}_2^{T^0\phi^0C^{-2}} = 2^{3/2}\xi_{1i}4k_{1c}k_{2a}\text{Tr}(\mathcal{C}'_{(n-1)}M_p\Gamma^{aic})\sqrt{\pi}\frac{\Gamma[-u+1/4]}{\Gamma[3/4-u]} \quad (21)$$

Therefore we might think of this term as the extra contact interaction to the S-matrix, however, after applying momentum conservation along the world volume of brane and using Bianchi identities<sup>19</sup>, we reveal the fact that this term has zero contribution to the S-matrix of an asymmetric RR, a scalar and a tachyon.<sup>20</sup>

## 6 The $\phi^0 - A^0 - C^{-2}$ amplitude

The four point function of an asymmetric RR, a scalar and a gauge field can be carried out by the following correlation function,

$$\mathcal{A}^{\phi^0 A^0 C^{-2}} \sim \int dx_1 dx_2 d^2 z \langle V_\phi^{(0)}(x_1) V_A^{(0)}(x_2) V_{RR}^{(-2)}(z, \bar{z}) \rangle \quad (22)$$

where the scalar field and RR vertex operators are given in the previous sections, and for the gauge field we have

$$V_A^{(0)}(x) = \xi_{2a}(\partial^a X(x) + i\alpha' k \cdot \psi \psi^a(x)) e^{\alpha' i k \cdot X(x)}$$

Having set the Wick theorem, the amplitude may have been written down as

$$\int dx_1 dx_2 dx_4 dx_5 (P_- \mathcal{C}'_{(n-1)} M_p)^{\alpha\beta} (x_{45})^{-3/4} \xi_{1i} \xi_{2a} (J_1 + J_2 + J_3 + J_4) I_5$$

and also

$$I_5 = |x_{12}|^{\alpha'^2 k_1 \cdot k_2} |x_{14} x_{15}|^{\frac{\alpha'^2}{2} k_1 \cdot p} |x_{24} x_{25}|^{\frac{\alpha'^2}{2} k_2 \cdot p} |x_{45}|^{\frac{\alpha'^2}{4} p \cdot D \cdot p}, \quad (23)$$

where by applying the generalization of Wick-like rule one can obtain all the correlators as follows

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<sup>19</sup>  $p^a \epsilon^{a_0 \dots a_{p-2} c a} = p^c \epsilon^{a_0 \dots a_{p-2} c a} = 0$

<sup>20</sup> It is important to point out that this term by itself without applying any Bianchi identity equation could be meant to be non-zero and might have been confused that it plays the role of the whole infinite contact interactions/ surface terms or total derivatives where clearly it does not play any contribution to the whole S-matrix.

$$\begin{aligned}
J_1 &= ip^i \frac{x_{54}}{x_{14}x_{15}} (l_a)(x_{45})^{-5/4} (C^{-1})_{\alpha\beta} \\
l_a &= -ik_{1a} \left[ \frac{x_{14}}{x_{12}x_{24}} + \frac{x_{15}}{x_{12}x_{25}} \right] \\
J_2 &= -p^i k_{2c} \frac{x_{54}}{x_{14}x_{15}} (x_{24}x_{25})^{-1} (x_{45})^{-1/4} (\Gamma^{ac} C^{-1})_{\alpha\beta} \\
J_3 &= (l_a) ik_{1b} (x_{14}x_{15})^{-1} (x_{45})^{-1/4} (\Gamma^{ib} C^{-1})_{\alpha\beta} \\
J_4 &= -k_{1b} k_{2c} (x_{14}x_{15}x_{24}x_{25})^{-1} (x_{45})^{3/4} \\
&\times \left[ (\Gamma^{acib} C^{-1})_{\alpha\beta} + (2\eta^{bc} (\Gamma^{ai} C^{-1})_{\alpha\beta} - 2\eta^{ab} (\Gamma^{ci} C^{-1})_{\alpha\beta}) \frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} \right] \quad (24)
\end{aligned}$$

If we now apply (24) into this four point amplitude we can easily determine that the S-matrix is  $SL(2, R)$  invariant. We do the proper gauge fixing as  $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$ , taking  $t = -\frac{\alpha'}{2}(k_1 + k_2)^2$  to actually get to the entire S-matrix as

$$\begin{aligned}
\mathcal{A}^{\phi^0 A^0 C^{-2}} &= -\xi_{1i} \xi_{2a} \int_{-\infty}^{\infty} dx (1+x^2)^{2t-1} (2x)^{-2t} \left[ \frac{1-x^2}{x} \left( -ip^i k_{1a} \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p) \right. \right. \\
&\quad \left. \left. + k_{1b} k_{1a} \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ib}) + \eta^{bc} \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ai}) - \eta^{ab} \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ci}) \right) \right. \\
&\quad \left. + 2ik_{2c} p^i \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ac}) - 2ik_{1b} k_{2c} \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{acib}) \right] \quad (25)
\end{aligned}$$

where the first, second, third and fourth term do not have any contribution to the S-matrix because the integration is taken on the whole space and the integrand is odd. If we use the momentum conservation  $(k_1 + k_2 + p)^a = 0$  and the Bianchi identity  $p_b \epsilon^{a_0 \dots a_{p-3} bac} = 0$ , then we come to the point that the sixth term has also no contribution to the amplitude so only the fifth term has non-zero contribution to the S-matrix of an asymmetric RR, a scalar and a gauge field.

Hence, the final result is

$$\mathcal{A}^{\phi^0 A^0 C^{-2}} = -\xi_{1i} \xi_{2a} 2ik_{2c} p^i \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ac}) \pi^{1/2} \frac{\Gamma[-t + 1/2]}{\Gamma[1-t]} \quad (26)$$

where the expansion of the amplitude is non-zero for  $p = n$  case and it does not include any poles as it is clear from the (26), because the low energy expansion is  $t \rightarrow 0$  limit and all the infinite contact interactions of this S-matrix have already been derived in [33]. Given the closed form of the above correlation functions one can find out to  $\mathcal{A}^{\phi^{-1} A^0 C^{-1}}$  as well



$$\begin{aligned} \mathcal{A}^{\phi^{-1}A^0C^{-1}} &= 2^{-3/2}\xi_{1i}\xi_{2a} \int_{-\infty}^{\infty} dx (1+x^2)^{2t-1} (2x)^{-2t} \left[ \frac{1-x^2}{x} \left( k_{1a} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^i) \right) \right. \\ &\quad \left. - 2ik_{2b} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \Gamma^{abi}) \right] \end{aligned}$$

where again the first term has no contribution and the second term (up to a coefficient of  $2^{3/2}$ ) precisely produces (26). Finally one explores this amplitude in its last picture as follows

$$\begin{aligned} \mathcal{A}^{\phi^0A^{-1}C^{-1}} &= 2^{1/2}\xi_{1i}\xi_{2a} \int_{-\infty}^{\infty} dx (1+x^2)^{2t-1} (2x)^{-2t} \left[ k_{1b} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \Gamma^{bai}) \right. \\ &\quad \left. - p^i \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^a) \right] \end{aligned} \quad (27)$$

where in (27), one has to apply momentum conservation to its first term and use the following Bianchi identity

$$p_b \epsilon^{a_0 \dots a_{p-2} ba} H_{a_0 \dots a_{p-2}}^i + p^i \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} = 0 \quad (28)$$

to actually get to the entire S-matrix as appeared in (26).

Therefore in this particular picture, again we are just left with one term for the final answer of the a RR, a gauge and a scalar field and this term is necessary because this S-matrix has to be produced by a non-zero coupling  $(2\pi\alpha')^2 \mu_p \int_{H^+} \partial_i C_{a_0 \dots a_{p-2}} F_{a_{p-1} a_p} \phi^i$  of effective field theory where the scalar field comes from Taylor expansion.

Note that by comparing this S-matrix with field theory, we come to understand that there should not be any other term in effective field theory coming from the pull-back of brane.

Now in order to obtain the other non trivial Bianchi identities, we are going to consider one of the most simplest five point functions and deal with more subtleties about perturbative string theory.

## 7 The five point world-sheet S-matrix of brane-anti brane system

It is known that the world volume of brane-anti brane system must have two real tachyon fields.<sup>21</sup> The complete form of the amplitude of a gauge ,two real tachyons and a closed string RR of brane anti brane for various  $p, n$  cases  $\langle V_A^{(-1)}(x_1)V_T^{(0)}(x_2)V_T^{(0)}(x_3)V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle$  has been derived in [41].

For the completeness we have done this amplitude in the following picture as well

$$\langle V_A^{(0)}(x_1)V_T^{(-1)}(x_2)V_T^{(0)}(x_3)V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle$$

and the final result for the amplitude is exactly the same as appeared in [41] so that it satisfies the Ward identity associated to the gauge field. It is also worth explaining that the S-matrix elements of one closed string RR field ,one scalar field and two tachyons on the world volume brane anti brane system in the following picture has also been computed in detail in [42]<sup>22</sup>

$$\langle V_\phi^{(-1)}(x_1)V_T^{(0)}(x_2)V_T^{(0)}(x_3)V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle \quad (31)$$

The final form of this S-matrix in this particular picture was given as

$$\mathcal{A}^{C^{(-1)}\phi^{(-1)}T^{(0)}T^{(0)}} = \mathcal{A}_1 + \mathcal{A}_2 \quad (32)$$

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<sup>21</sup> In [40] it is discussed in detail that brane anti brane system should be investigated by means of the effective field theory techniques which seems to be the proper way of realising classical or even for loop divergences for which anti brane dynamics in the presence of some of background fields may play a major role.

<sup>22</sup> The following vertices with their correct Chan-Paton factors of D-brane anti D-brane are held

$$\begin{aligned} V_\phi^{(-1)}(x) &= \xi_i \psi^i(x) e^{2iq \cdot X(x)} e^{-\phi(x)} \otimes \sigma_3 \\ V_T^{(0)}(y) &= 2ik \cdot \psi(y) e^{2ik \cdot X(y)}, \otimes \sigma_1 \\ V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{ip \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{ip \cdot D \cdot X(\bar{z})} \otimes \sigma_3, \end{aligned} \quad (29)$$

so that  $k^2 = 1/4$  is the condition for tachyons in type II string theory and the following definitions for Mandelstam variables are used

$$s = -\frac{\alpha'}{2}(k_1 + k_3)^2, t = -\frac{\alpha'}{2}(k_1 + k_2)^2, u = -\frac{\alpha'}{2}(k_2 + k_3)^2, \quad (30)$$

where

$$\begin{aligned}\mathcal{A}_1 &\sim -8\xi_{1i}k_{2a}k_{3b}2^{-3/2}\text{Tr}(P_- \mathbb{H}_{(n)} M_p \Gamma^{bai}) L_1, \\ \mathcal{A}_2 &\sim 8\xi_{1i}2^{-3/2} \left\{ \text{Tr}(P_- \mathbb{H}_{(n)} M_p \gamma^i) \right\} L_2\end{aligned}\quad (33)$$

where  $L_1, L_2$  are written down in below just in terms of Gamma functions ( no hypergeometric function is needed)

$$\begin{aligned}L_1 &= (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u)\Gamma(-s+\frac{1}{4})\Gamma(-t+\frac{1}{4})\Gamma(-t-s-u)}{\Gamma(-u-t+\frac{1}{4})\Gamma(-t-s+\frac{1}{2})\Gamma(-s-u+\frac{1}{4})}, \\ L_2 &= (2)^{-2(t+s+u+1)} \pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{3}{4})\Gamma(-t+\frac{3}{4})\Gamma(-t-s-u-\frac{1}{2})}{\Gamma(-u-t+\frac{1}{4})\Gamma(-t-s+\frac{1}{2})\Gamma(-s-u+\frac{1}{4})}\end{aligned}\quad (34)$$

where in [42] all the infinite  $u$ - channel gauge poles of  $L_1$  and  $t+s'+u'$  channel scalar poles of this S-matrix have been precisely produced, in addition to that all the infinite higher derivative corrections to two scalars-two tachyons of world volume of brane anti brane without any ambiguity have also been discovered.

Now let us deal with this S-matrix in the other pictures of both closed-open strings to see what happens to the complete form of the S-matrix and also explore all its contact interactions.

## 8 $C^{(-2)}\phi^{(0)}T^{(0)}T^{(0)}$

The S-matrix element of an asymmetric closed string RR field, one scalar field and two tachyons on the world volume of brane anti brane system can be found as follows. <sup>23</sup> Replacing the vertex operators and performing all the correlators by using the Wick theorem, one can investigate to obtain the whole amplitude as follows

$$\begin{aligned}\mathcal{A}^{C^{(-2)}\phi^{(0)}T^{(0)}T^{(0)}} &\sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} \xi_{1i} x_{45}^{-3/4} (-8k_{2b}k_{3c}) I \\ &\times \left( (ip^i \frac{x_{54}}{x_{15}x_{14}}) I_1^{cb} + 2ik_{1a} I_2^{cbia} \right)\end{aligned}\quad (36)$$

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<sup>23</sup> The following vertices with their correct Chan-Paton factor for D-brane-anti D-brane are taken into account ,

$$\begin{aligned}V_\phi^{(0)}(x) &= \xi_{1i}(\partial^i X(x) + i\alpha' k \cdot \psi \psi^i(x)) e^{\alpha' ik \cdot X(x)} \lambda \otimes I \\ V_T^{(0)}(y) &= 2ik \cdot \psi(y) e^{2ik \cdot X(y)}, \otimes \sigma_1 \\ V_{RR}^{(-\frac{3}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} e^{-3\phi(z)/2} S_\alpha(z) e^{ip \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{ip \cdot D \cdot X(\bar{z})} \otimes I,\end{aligned}\quad (35)$$

with the same definitions for Mandelstam variables kept held.

where  $x_{ij} = x_i - x_j$ ,

$$I = |x_{12}|^{4k_1 \cdot k_2} |x_{13}|^{4k_1 \cdot k_3} |x_{14}x_{15}|^{2k_1 \cdot p} |x_{23}|^{4k_2 \cdot k_3} |x_{24}x_{25}|^{2k_2 \cdot p} |x_{34}x_{35}|^{2k_3 \cdot p} |x_{45}|^{p \cdot D \cdot p} \quad (37)$$

Note that we have already generalised the Wick theorem to get to the fermionic correlations in the presence of currents so that one can find the following correlators as

$$I_1^{cb} = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^b(x_2) :: \psi^c(x_3) : \rangle = 2^{-1} x_{45}^{-1/4} (x_{24}x_{25}x_{34}x_{35})^{-1/2} \\ \times \left( (\Gamma^{cb} C^{-1})_{\alpha\beta} - 2\eta^{bc} \frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} (C^{-1})_{\alpha\beta} \right)$$

also

$$I_2^{cbia} = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a \psi^i(x_1) : \psi^b(x_2) :: \psi^c(x_3) : \rangle = 2^{-2} x_{45}^{3/4} (x_{24}x_{25}x_{34}x_{35})^{-1/2} (x_{14}x_{15})^{-1} \\ \left\{ (\Gamma^{cbia} C^{-1})_{\alpha\beta} + 2\eta^{ab} \frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} (\Gamma^{ci} C^{-1})_{\alpha\beta} - 2\eta^{ac} \frac{Re[x_{14}x_{35}]}{x_{13}x_{45}} (\Gamma^{bi} C^{-1})_{\alpha\beta} \right. \\ \left. - 2\eta^{bc} \frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} (\Gamma^{ia} C^{-1})_{\alpha\beta} \right\}$$

If we substitute all the above correlations inside the amplitude, then one can show that the property of  $SL(2, \mathbb{R})$  invariance is being investigated. Here we are working with five point function and to our knowledge the best gauge fixing for this amplitude is to fix the locations of all three open strings as follows

$$x_1 = 0, \quad x_2 = 1, \quad x_3 \rightarrow \infty,$$

If we do so, then we get the entire form of the S-matrix in terms of some integrations on the upper half plane so that the following integrations for various cases need to be performed

$$\int d^2z |1 - z|^a |z|^b (z - \bar{z})^c (z + \bar{z})^d, \quad (38)$$

with  $d = 0, 1, 2$  and  $a, b, c$  must be just given in terms of the Mandelstam variables. Notice to the point that the result of the above integrations for  $d = 0, 1$  is got from [43, 41] while for  $d = 2$  the result is given in [13].

If we do gauge fixing, make use of various pure algebraic simplifications and most particularly make use of the results of the integrals that pointed out in [43, 41, 13] then we can write down the final result of the amplitude (36) in this asymmetric picture as follows

$$\mathcal{A}^{C^{-2}\phi^0 T^0 T^0} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 \quad (39)$$

where

$$\begin{aligned} \mathcal{A}_1 &\sim ip^i \xi_{1i} (4k_{2b} k_{3c}) \text{Tr} (P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{cb}) L_1, \\ \mathcal{A}_2 &\sim 4ip^i \xi_{1i} \left\{ \text{Tr} (P_- \mathcal{C}'_{(n-1)} M_p) \right\} L_2 \\ \mathcal{A}_3 &\sim -4i \xi_{1i} k_{1a} k_{2b} k_{3c} \left\{ \text{Tr} (P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{cbia}) \right\} L_1 \\ \mathcal{A}_4 &\sim 4i \xi_{1i} \left\{ \text{Tr} (P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{bi}) \right\} L_2 (k_{1b} + k_{2b} + k_{3b}) \end{aligned} \quad (40)$$

where the functions  $L_1, L_2$  are given in (34).

Note that here in (40) we have dealt with the five point world sheet scattering of brane anti brane in asymmetric picture and in this particular picture we found the terms that carry momentum of RR in transverse directions. These terms are no longer vanished and indeed these terms potentially have something to do with some of the issues on the perturbative string theory on upper half plane . We believe that these terms are related to taking integration on different moduli space that have been pointed out in series of papers appeared in [34].

Now if we compare the S-matrix in this asymmetric picture (40) with (33) , then we might think of the fact that the last two terms of (40) are extra singularities. Indeed if we do not apply the Bianchi identity and momentum conservation along the brane to (40) as a matter of fact these two terms would be extra terms by themselves. However, if we compare (40) with (33), simultaneously extract the trace in the last term of (40) and make use of the momentum conservation along the world volume of brane as follows

$$(k_1 + k_2 + k_3)^a = -p^a \quad (41)$$

we come to the conclusion that the last term in (40) is apparent singularity and this should be removed, in the other words upon applying the following Bianchi identity

$$p^b \epsilon^{a_0 \dots a_{p-1} b} C_{a_0 \dots a_{p-1}}^i = 0 \quad (42)$$

the last term of (40) vanishes.

Note that in below we show that the above Bianchi identity can not be applied to the correlators

$$\langle V_T^{(0)}(x_1)V_T^{(0)}(x_2)V_T^{(0)}(x_3)V_{RR}^{(-\frac{3}{2},-\frac{1}{2})}(z,\bar{z})\rangle$$

of non-BPS branes. Indeed after gauge fixing as  $(x_1, x_2, z, \bar{z}) = (0, 1, \infty, z, \bar{z})$  we gain the following non-BPS amplitude

$$\begin{aligned} \mathcal{A}^{C^{-2}T^0T^0T^0} = & -16i(2^{-3/2}k_{1a}k_{2b}k_{3c})(P_- \mathcal{C}_{(n)} M_p)^{\alpha\beta} \int d^2z |1-z|^{2t+2u} |z|^{2t+2s} (z-\bar{z})^{-2(t+s+u+1)} \\ & \times \left[ (\Gamma^{cba} C^{-1})_{\alpha\beta} + \frac{(z+\bar{z})}{2(z-\bar{z})} \left( (2\eta^{ac}(\gamma^b C^{-1})_{\alpha\beta}) - (2\eta^{ab}(\gamma^c C^{-1})_{\alpha\beta}) - (2\eta^{bc}(\gamma^a C^{-1})_{\alpha\beta}) \right) \right. \\ & \left. + (2\eta^{bc}(\gamma^a C^{-1})_{\alpha\beta}) \frac{1}{(z-\bar{z})} + (2\eta^{ab}(\gamma^c C^{-1})_{\alpha\beta}) \frac{|z|^2}{(z-\bar{z})} \right] \end{aligned}$$

We make use of various pure algebraic simplifications to write down the final result for the above amplitude in this asymmetric picture as follows

$$\mathcal{A}^{C^{-2}T^0T^0T^0} = \mathcal{A}_1 + \mathcal{A}_2$$

where

$$\begin{aligned} \mathcal{A}_1 & \sim -16i(2^{-3/2}k_{1a}k_{2b}k_{3c})\text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{cba}) N_1 \\ \mathcal{A}_2 & \sim -16i(2^{-3/2})\text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \gamma^a) N_2 (k_{1a} + k_{2a} + k_{3a}) \end{aligned}$$

where the functions  $N_1, N_2$  are given as

$$\begin{aligned} N_1 &= (2)^{-2(t+s+u+1)} \pi \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u-\frac{1}{2})}{\Gamma(-u-t)\Gamma(-t-s)\Gamma(-s-u)}, \\ N_2 &= (2)^{-2(t+s+u)-3} \pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u-1)}{\Gamma(-u-t)\Gamma(-t-s)\Gamma(-s-u)} \end{aligned} \tag{43}$$

Note that if we use momentum conservation  $p^a = -(k_1 + k_2 + k_3)^a$  for both first and second part of the above non-BPS S-matrix we get

$$p^a \epsilon^{a_0 \dots a_{p-3} cba} C_{a_0 \dots a_{p-3}}, p^c \epsilon^{a_0 \dots a_{p-3} cba} C_{a_0 \dots a_{p-3}}, p^a \epsilon^{a_0 \dots a_{p-1} a} C_{a_0 \dots a_{p-1}}$$

are not zero. In particular in order to make sense of non supersymmetric amplitudes in the world volume of non-BPS branes, the following equations

$$p^a \epsilon^{a_0 \dots a_{p-3} cba}, \quad p^a \epsilon^{a_0 \dots a_{p-1} a} \quad (44)$$

must be non zero.

Hence for non super symmetric amplitudes first we must extract the traces and keep in mind the above points , that is , the equations that we found for some of the BPS branes, can not be applied to non-BPS amplitudes in the presence of scalar field and closed string RR. Indeed we expect to see that behaviour because the equations that hold for BPS cases not necessary can be held for non supersymmetric cases whereas for BPS branes the equations seem to be more manifest while this may have been changed after symmetry breaking. The lesson is that for scattering of the mixed scalars- tachyons in the presence of RR ( in its asymmetric picture), one needs to break several identities that necessary hold for BPS branes.

What about the third term of (40)? One might add it with the first term of (40) and come to the point that

$$k_{2b} k_{3c} \xi_{1i} (p^i \epsilon^{a_0 \dots a_{p-2} cb} C_{a_0 \dots a_{p-2}} + p^a \epsilon^{a_0 \dots a_{p-3} cba} C_{a_0 \dots a_{p-3}}^i)$$

should be vanished, however from (33) we know that the first term of (40) holds and plays the crucial role in effective field theory. Thus we need to explore new Bianchi identity for the third term of (40). In fact if we actually apply momentum conservation to the third term of (40) and because of the antisymmetric property of  $\epsilon$  we conclude that the equation  $p^a \epsilon^{a_0 \dots a_{p-3} cba} C_{a_0 \dots a_{p-3}}^i$  must be vanished for brane anti brane amplitudes.

Therefore, the lesson we have got is as follows. In the presence of an asymmetric RR , a scalar and some tachyons, one needs to find out new Bianchi identities to get to the same S-matrix elements as obtained by a symmetric RR , a scalar and some tachyons. Because we have no gauge field to check the gauge invariance of the amplitude and more accurately there is no Ward identity for the scalar field .

Now let us come over to the same S-matrix of a symmetric RR, two tachyons and a

scalar field in zero picture  $< C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)} > .$

## 9 $C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)}$

Finally this S-matrix element of a symmetric RR field ,one scalar in zero picture and two tachyons on the world volume of brane anti brane system can be written down by

$$\mathcal{A}^{C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)}} \sim \int dx_1 dx_2 dx_3 d^2 z \langle V_\phi^{(0)}(x_1) V_T^{(-1)}(x_2) V_T^{(0)}(x_3) V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle$$

with the following vertices

$$\begin{aligned} V_T^{(-1)}(y) &= e^{-\phi(y)} e^{2ik \cdot X(y)} \otimes \sigma_2 \\ V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{ip \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{ip \cdot D \cdot X(\bar{z})} \otimes \sigma_3, \end{aligned} \quad (45)$$

we just write down the amplitude in its compact form as

$$\begin{aligned} \mathcal{A}^{C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)}} &\sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \xi_{1i}(4k_{3b}) x_{45}^{-1/4} (x_{24} x_{25})^{-1/2} I \\ &\times \left( (ip^i \frac{x_{54}}{x_{15} x_{14}}) I_1^b + 2ik_{1a} I_2^{bia} \right) \end{aligned}$$

where  $x_{ij} = x_i - x_j$ ,

$$I = |x_{12}|^{4k_1 \cdot k_2} |x_{13}|^{4k_1 \cdot k_3} |x_{14} x_{15}|^{2k_1 \cdot p} |x_{23}|^{4k_2 \cdot k_3} |x_{24} x_{25}|^{2k_2 \cdot p} |x_{34} x_{35}|^{2k_3 \cdot p} |x_{45}|^{p \cdot D \cdot p}$$

where the following correlators need to be replaced inside the amplitude

$$I_1^b = <: S_\alpha(x_4) : S_\beta(x_5) : \psi^b(x_3) :> = 2^{-1/2} x_{45}^{-3/4} (x_{34} x_{35})^{-1/2} (\gamma^b C^{-1})_{\alpha\beta}$$

also

$$\begin{aligned} I_2^{bia} &= <: S_\alpha(x_4) : S_\beta(x_5) : \psi^a \psi^i(x_1) : \psi^b(x_3) :> = 2^{-3/2} x_{45}^{1/4} (x_{34} x_{35})^{-1/2} (x_{14} x_{15})^{-1} \\ &\left\{ (\Gamma^{bia} C^{-1})_{\alpha\beta} + 2\eta^{ab} \frac{Re[x_{14} x_{35}]}{x_{13} x_{45}} (\gamma^i C^{-1})_{\alpha\beta} \right\} \end{aligned}$$

Notice that, given the above correlations , one may easily investigate the SL(2,R) transformation of the S-matrix. We performed gauge fixing by just fixing the location of open



strings so that the final integration needs to be done over the closed string RR 's position on the upper half complex plane. <sup>24</sup>

Once more after gauge fixing one finds the same sort of the integration as we discussed earlier on. <sup>25</sup>

Having calculated all the desired integrals, one could explore the result for the S-matrix in its particular picture as follows

$$\mathcal{A}^{C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)}} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 \quad (47)$$

in such a way that all its components are given by

$$\begin{aligned} \mathcal{A}_1 &\sim -2i2^{1/2}p^i\xi_{1i}k_{3b}\text{Tr}(P_-H_{(n)}M_p\gamma^b)L_1, \\ \mathcal{A}_2 &\sim -2i2^{1/2}\xi_{1i}\left\{\text{Tr}(P_-H_{(n)}M_p\Gamma^{bia})\right\}k_{1a}k_{3b}L_1 \\ \mathcal{A}_3 &\sim -2i2^{1/2}\xi_{1i}\left\{\text{Tr}(P_-H_{(n)}M_p\gamma^i)\right\}L_2 \end{aligned} \quad (48)$$

where  $L_1, L_2$  functions are already given in (34). If we compare (48) with (33), then one might wonder whether the first term of  $\mathcal{A}^{C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)}}$  is extra term, because it is also related to all infinite singularities. However, it is worth pointing that we have already produced all the infinite u-channel gauge poles by taking into account the second term of (48). Therefore one has to find out or generalise new Bianchi identities to actually remove the first extra term of (48). Hence we apply momentum conservation  $k_{1a} = -k_{2a} - k_{3a} - p_a$  to the second term of (48), extract the traces, use the antisymmetric property of  $\epsilon$  tensor and eventually add the first and second components of the amplitude to derive the following Bianchi identity

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<sup>24</sup>

$$x_1 = 0, \quad x_2 = 1, \quad x_3 \rightarrow \infty, \quad dx_1 dx_2 dx_3 \rightarrow x_3^2.$$

<sup>25</sup>

$$\int d^2z |1 - z|^a |z|^b (z - \bar{z})^c (z + \bar{z})^d, \quad (46)$$

$$\xi_{1i}k_{3b}\left(-p_a\epsilon^{a_0\cdots a_{p-2}ab}H_{a_0\cdots a_{p-2}}^i+p^i\epsilon^{a_0\cdots a_{p-1}b}H_{a_0\cdots a_{p-1}}\right)=0 \quad (49)$$

Thus by holding (49) and keeping in mind momentum conservation along the world volume of brane we are precisely able to obtain the same S-matrix as appeared in (33), so we come to the crucial fact about this five point function of brane anti brane system as follows.

If we consider closed string RR in the asymmetric picture  $C^{-3/2,-1/2}$  and scalar field in zero picture with some other open string tachyons then one must find out all new Bianchi identities to be able to remove all the extra apparent singularities of asymmetric picture of brane/ anti brane systems.

In the next section we will reveal some remarks for non-BPS branes to actually restore Ward identity associated (gauge invariance) to the gauge field and also we derive all the precise contact interactions of mixed RR, scalar/ gauge and tachyon string amplitudes. Namely we show that if we consider the scalar field in zero picture and the RR in asymmetric picture ( $C^{-3/2,-1/2}$ ) then in this particular case there is no need to explore new Bianchi identities to the S-matrix of non-BPS amplitudes and more importantly those amplitudes independently respect the Ward identity associated to the gauge field.

## 10 $C^{-1}\phi^0A^{-1}T^0$ S-matrix

In [35] the five point world-sheet amplitude of a symmetric closed string RR with one scalar, a gauge field and a tachyon in the world volume of non-BPS branes of type II string theory  $< C^{-1}\phi^{-1}A^0T^0 >$  was achieved and all the correlators were found.<sup>26</sup>

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26

$$\mathcal{A}^{C^{-1}\phi^{-1}A^0T^0} \sim \int dx_1dx_2dx_3dzd\bar{z} \langle V_\phi^{(-1)}(x_1)V_A^{(0)}(x_2)V_T^{(0)}(x_3)V_{RR}^{(-\frac{1}{2},-\frac{1}{2})}(z,\bar{z}) \rangle, \quad (50)$$

with the following vertex operators :

$$\begin{aligned} V_\phi^{(-1)}(y) &= \xi_{1i}\psi^i(y)e^{\alpha'ik.X(y)}e^{-\phi(y)}\otimes\sigma_3 \\ V_A^{(0)}(x) &= \xi_{2a}(\partial^aX(x)+i\alpha'q.\psi\psi^a(x))e^{\alpha'iq.X(x)}\otimes I \end{aligned}$$

The final result in this symmetric picture was read off as

$$\mathcal{A}^{C^{-1}\phi^{-1}A^0T^0} = \mathcal{A}_1 + \mathcal{A}_2 \quad (51)$$

where

$$\begin{aligned} \mathcal{A}_1 &\sim 2\xi_{1i}\xi_{2a}k_{3c}k_{2d}\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{cadi}) L'_1, \\ \mathcal{A}_2 &\sim (2L'_3) \left\{ -t\text{Tr}(P_- \not{H}_{(n)} M_p \gamma \cdot \xi_2 \gamma \cdot \xi_1) u' - 2tk_3 \cdot \xi_2 \text{Tr}(P_- \not{H}_{(n)} M_p \gamma \cdot k_2 \gamma \cdot \xi_1) \right. \\ &\quad \left. + \text{Tr}(P_- \not{H}_{(n)} M_p \gamma \cdot k_3 \gamma \cdot \xi_1) \left( -2t(k_3 \cdot \xi_2) + 2u'k_1 \cdot \xi_2 \right) \right\} \end{aligned} \quad (52)$$

It was also shown that one needs to use the momentum conservation as  $s + t + u = -p^a p_a - \frac{1}{4}$ , applying  $t \rightarrow 0$ ,  $s \rightarrow -\frac{1}{4}$ ,  $u \rightarrow -\frac{1}{4}$  to the S-matrix to be able to derive all infinite  $u', t$  channel tachyon and scalar poles of the non super symmetric amplitudes accordingly. Note that  $L'_1$  has just infinite contact interactions.

It is of high importance to note that this particular  $\mathcal{A}^{C^{-1}\phi^{-1}A^0T^0}$  amplitude does respect all the symmetries and most importantly it does satisfy Ward identity associated to the gauge field. Indeed if we replace  $\xi_{2a} \rightarrow k_{2a}$  inside (52) then the first term of (52) is automatically zero because

$$k_{2a}k_{2d}k_{3c}\epsilon^{a_0 \dots a_{p-2}cad} = 0$$

replacing  $\xi_{2a} \rightarrow k_{2a}$  inside the second, third, fourth and fifth term of (52) we also get zero result for the whole S-matrix <sup>27</sup> so gauge invariance is satisfied. Thus based on just replacing  $\xi_{2a} \rightarrow k_{2a}$  of the gauge field, the whole S-matrix vanishes.

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and  $u' = (-u - \frac{1}{4})$  and also

$$\begin{aligned} L'_1 &= (2)^{-2(t+s+u)} \pi \frac{\Gamma(-u + \frac{1}{4})\Gamma(-s + \frac{1}{4})\Gamma(-t + \frac{1}{2})\Gamma(-t - s - u + \frac{1}{2})}{\Gamma(-u - t + \frac{3}{4})\Gamma(-t - s + \frac{3}{4})\Gamma(-s - u + \frac{1}{2})}, \\ L'_3 &= (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u - \frac{1}{4})\Gamma(-s + \frac{3}{4})\Gamma(-t)\Gamma(-t - s - u)}{\Gamma(-u - t + \frac{3}{4})\Gamma(-t - s + \frac{3}{4})\Gamma(-s - u + \frac{1}{2})} \end{aligned}$$

<sup>27</sup>

$$k_{2a}\xi_{1i}(u' - u')\text{Tr}(P_- \not{H}_{(n)} M_p \gamma^a \gamma^i) = 0$$

$$k_{3a}\xi_{1i}(tu' - tu')\text{Tr}(P_- \not{H}_{(n)} M_p \gamma^a \gamma^i) = 0$$

Having explained all the needed ingredients of the S-matrices , in the following we would like to change the vertex of scalar field and see what happens to the gauge invariance of a mixture of five point world-sheet amplitude of a symmetric closed string RR with one scalar in zero picture , a gauge field and a tachyon in the world volume of non-BPS branes of type II string theory . This  $< C^{-1}\phi^0 A^{-1}T^0 >$  amplitude is given by the following correlation functions

$$\mathcal{A}^{C^{-1}\phi^0 A^{-1}T^0} \sim \int dx_1 dx_2 dx_3 dz d\bar{z} \langle V_\phi^{(0)}(x_1) V_A^{(-1)}(x_2) V_T^{(0)}(x_3) V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle, \quad (53)$$

Let us write down the rest of the vertex operators including their CP factors as

$$\begin{aligned} V_A^{(-1)}(y) &= \xi_{2a} \psi^a(y) e^{\alpha' i q \cdot X(y)} e^{-\phi(y)} \otimes \sigma_3 \\ V_\phi^{(0)}(x) &= \xi_{1i} (\partial^i X(x) + i \alpha' k \cdot \psi \psi^i(x)) e^{\alpha' i k \cdot X(x)} \otimes I \end{aligned} \quad (54)$$

where the following on-shell conditions for scalar, gauge, RR and tachyon hold.<sup>28</sup> Once more we deal with just the holomorphic elements of all fields involving  $X^\mu \psi^\mu, \phi$ , so that the S-matrix is now given by

$$\begin{aligned} \mathcal{A}^{C^{-1}\phi^0 A^{-1}T^0} &\sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \xi_{1i} \xi_{2a} (\alpha' i k_{3c}) x_{45}^{-1/4} (x_{24} x_{25})^{-1/2} \\ &\times (I_1 + I_2) I \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \text{Tr}(\sigma_3 \sigma_1 I \sigma_3 \sigma_1), \end{aligned} \quad (55)$$

where<sup>29</sup>  $x_{ij} = x_i - x_j$ . Let us find out all the fermionic and bosonic correlators as

$$\begin{aligned} I_1 &= \langle : \partial X^i(x_1) e^{\alpha' i k_1 \cdot X(x_1)} : e^{\alpha' i k_2 \cdot X(x_2)} : e^{\alpha' i k_3 \cdot X(x_3)} : e^{\frac{\alpha'}{2} i p \cdot X(x_4)} : e^{\frac{\alpha'}{2} i p \cdot D \cdot X(x_5)} : \rangle \\ &\times \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_2) : \psi^c(x_3) \rangle, \\ I_2 &= \langle : e^{\alpha' i k_1 \cdot X(x_1)} : e^{\alpha' i k_2 \cdot X(x_2)} : e^{\alpha' i k_3 \cdot X(x_3)} : e^{\frac{\alpha'}{2} i p \cdot X(x_4)} : e^{\frac{\alpha'}{2} i p \cdot D \cdot X(x_5)} : \rangle \\ &\alpha' i k_{1d} \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^d \psi^i(x_1) : \psi^a(x_2) : \psi^c(x_3) : \rangle \end{aligned} \quad (57)$$

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<sup>28</sup>

$$k^2 = q^2 = p^2 = 0, \quad k_3^2 = 1/4, \quad q \cdot \xi_1 = k_2 \cdot \xi_1 = 0$$

<sup>29</sup>

$$I = |x_{12}|^{\alpha'^2 k_1 \cdot k_2} |x_{13}|^{\alpha'^2 k_1 \cdot k_3} |x_{14} x_{15}|^{\frac{\alpha'^2}{2} k_1 \cdot p} |x_{23}|^{\alpha'^2 k_2 \cdot k_3} |x_{24} x_{25}|^{\frac{\alpha'^2}{2} k_2 \cdot p} |x_{34} x_{35}|^{\frac{\alpha'^2}{2} k_3 \cdot p} |x_{45}|^{\frac{\alpha'^2}{4} p \cdot D \cdot p} \quad (56)$$

We need to use the Wick-like rule [37] to get to all the generalizations of the correlation functions of two spin and two fermion operators such as the following

$$I_5^{ca} = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_2) : \psi^c(x_3) : \rangle = 2^{-1} x_{45}^{-1/4} (x_{24} x_{25} x_{34} x_{35})^{-1/2} \\ \times \left\{ (\Gamma^{ca} C^{-1})_{\alpha\beta} - 2 \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} \eta^{ac} (C^{-1})_{\alpha\beta} \right\}$$

Now we need to make use of [12] to come over the final answer of the correlation function in ten dimensions,

$$I_6^{caid} = \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^d \psi^i(x_1) : \psi^a(x_2) : \psi^c(x_3) : \rangle \\ = \left\{ (\Gamma^{caid} C^{-1})_{\alpha\beta} + \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} (2\eta^{da} (\Gamma^{ci} C^{-1})_{\alpha\beta}) - 2 \frac{Re[x_{14} x_{35}]}{x_{13} x_{45}} \eta^{dc} (\Gamma^{ai} C^{-1})_{\alpha\beta} \right. \\ \left. - 2 \frac{Re[x_{24} x_{35}]}{x_{23} x_{45}} \eta^{ac} (\Gamma^{id} C^{-1})_{\alpha\beta} \right\} 2^{-2} x_{45}^{3/4} (x_{24} x_{25} x_{34} x_{35})^{-1/2} (x_{14} x_{15})^{-1}, \quad (58)$$

Now we are allowed to replace all the correlators inside (55) to obtain the compact form of the desired S-matrix as follows

$$\mathcal{A}^{C^{-1}\phi^0 A^{-1}T^0} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- \mathcal{H}_{(n)} M_p)^{\alpha\beta} I_{\xi_1 i \xi_{2a}} (-2\alpha' i k_{3c}) x_{45}^{-1/4} (x_{24} x_{25})^{-1/2} \\ \times \left( a_2^i I_5^{ca} + \alpha' i k_{1d} I_6^{caid} \right) \text{Tr}(\lambda_1 \lambda_2 \lambda_3), \quad (59)$$

so that

$$a_2^i = i p^i \left( \frac{x_{54}}{x_{14} x_{15}} \right) \quad (60)$$

We are now able to investigate that the amplitude has satisfied the  $SL(2, R)$  invariance, we also did gauge fixing by just fixing the positions of three open strings.<sup>30</sup>

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$$x_1 = 0, \quad x_2 = 1, \quad x_3 \rightarrow \infty, \quad (61)$$

we lead to

$$\int d^2 z |1 - z|^a |z|^b (z - \bar{z})^c (z + \bar{z})^d, \quad (62)$$

with following Mandelstam variables

$$s = -\frac{\alpha'}{2} (k_1 + k_3)^2, \quad t = -\frac{\alpha'}{2} (k_1 + k_2)^2, \quad u = -\frac{\alpha'}{2} (k_2 + k_3)^2.$$

see [43]

The solutions for all the integrals on upper half plane have been released and the ultimate result of the S-matrix will be obtained as

$$\mathcal{A}^{C^{-1}\phi^0 A^{-1}T^0} = \mathcal{A}_1 + \mathcal{A}_2, \quad (63)$$

where

$$\begin{aligned} \mathcal{A}_1 &\sim \left( 2\xi_{1i}\xi_{2a}k_{3c}k_{1d}\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{caid}) - \xi_1.p(2k_{3c}\xi_{2a})\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{ca}) \right) L'_1, \\ \mathcal{A}_2 &\sim \left\{ t\xi_1.p(4k_3.\xi_2)\text{Tr}(P_- \not{H}_{(n)} M_p) + 4(u + \frac{1}{4})k_{3c}\xi_{1i}\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{ci})k_1.\xi_2 \right. \\ &\quad \left. - 4tk_3.\xi_2k_{1b}\xi_{1i}\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{ib}) - 2t(u + \frac{1}{4})\xi_{1i}\xi_{2a}\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{ai}) \right\} L'_3 \end{aligned} \quad (64)$$

Note that we have already analysed all infinite  $u'$  tachyon and massless  $t$  channel scalar poles of the amplitude in [35].

In this picture after replacing  $\xi_{2a} \rightarrow k_{2a}$  ( due to the terms  $\xi_1.p$  ) the amplitude does not satisfy Ward identity associated to the gauge field and indeed the second and third terms would remain whereas  $L'_3$  can not be removed by  $L'_1$ . Let us compare the result of this amplitude (63) with (52) to make a statement on mixed closed and open string amplitudes including one scalar and one RR and some other open tachyons. The last term in (64) is exactly the second term of (52), the fourth term in (64) is exactly the last term of (52). Note that if we add the third and fourth term of (52) and use the momentum conservation along the world volume of brane , then the result is precisely equivalent with the fifth term of (64). Once more by using momentum conservation in world volume direction the first term of (64) is exactly equivalent with the first term in (52).

However, the second and the third terms of (64) are extra terms and in particular if we use anti commutator relation of  $\gamma$  matrices these two terms can not cancel each other due to the fact that  $L'_3$  is different from  $L'_1$  .

Indeed if we replace  $\xi_{2a} \rightarrow k_{2a}$  inside (64) and use momentum conservation along the brane , the first term is automatically zero because

$$k_{2a}k_{3c}k_{1d}\epsilon^{a_0 \dots a_{p-2} cad} = 0$$

By replacing  $\xi_{2a} \rightarrow k_{2a}$  inside the fourth, fifth and sixth terms of (64) appropriately we also get zero result as follows

$$- 2tu'\xi_{1i}(k_{2a} + k_{1a} + k_{3a})\epsilon^{a_0 \dots a_{p-1} a} = 0$$

Thus the second and third terms give rise the amplitude not to be gauge invariant unless one finds out some new Bianchi identities.<sup>31</sup>

In the next section we show that by considering the asymmetric RR and a scalar, a gauge and one tachyon in zero picture of non-BPS branes, the S-matrix automatically satisfies Ward identity without needing to introduce any new Bianchi identities.

## 10.1 $C^{-2}\phi^0 A^0 T^0$ S-matrix

One can do the same CFT methods to actually derive the entire S-matrix of the above strings in the asymmetric picture. Hence the final answer for the five point world-sheet amplitude of a closed string RR (in asymmetric picture) with one scalar, a gauge field and a tachyon in the world volume of non-BPS branes of type II string theory  $\langle C^{-2}\phi^0 A^0 T^0 \rangle$  is

$$\mathcal{A}^{C^{-2}\phi^0 A^0 T^0} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 \quad (66)$$

where

$$\begin{aligned} \mathcal{A}_1 &\sim 2^{3/2} i \xi_{1i} \xi_{2a} k_{3c} k_{2b} L'_1 \left( p^i \text{Tr} (P_- \mathcal{C}'_{(n)} M_p \Gamma^{cab}) - k_{1d} \text{Tr} (P_- \mathcal{C}'_{(n)} M_p \Gamma^{cabid}) \right) \\ \mathcal{A}_2 &\sim 2^{3/2} i \xi_{1i} \cdot p L'_3 \text{Tr} (P_- \mathcal{C}'_{(n)} M_p \gamma^c) \left( 2t k_3 \cdot \xi_2 [-k_{3c} - k_{2c}] + 2k_1 \cdot \xi_2 u' k_{3c} - t u' \xi_{2c} \right) \\ \mathcal{A}_3 &\sim 2^{3/2} i \xi_{1i} L'_3 \text{Tr} (P_- \mathcal{C}'_{(n)} M_p \Gamma^{cid}) \left[ -2k_1 \cdot \xi_2 u' k_{3c} (k_{1d} + k_{2d}) + 2t k_3 \cdot \xi_2 k_{1d} (k_{3c} + k_{2c}) \right] \\ \mathcal{A}_4 &\sim 2^{3/2} i \xi_{1i} L'_3 t u' \xi_{2a} \text{Tr} (P_- \mathcal{C}'_{(n)} M_p \Gamma^{cai}) (k_{3c} + k_{1c} + k_{2c}) \end{aligned} \quad (67)$$

with  $u' = (-u - \frac{1}{4})$  and the same introduced  $L'_1, L'_3$ .  $L'_1$  has infinite contact interactions and  $L'_3$  has infinite  $t, u'$  scalar-tachyon channel poles accordingly.

The nice thing about this asymmetric S-matrix is that without introducing any further Bianchi identity this amplitude automatically does satisfy the Ward identity associated

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<sup>31</sup>The resolution for this problem (to get satisfied gauge invariance of the above S-matrix) is to add up the third term of (64) with the other terms in  $\mathcal{A}_2$  of (64) and also do add the first and second term of (64) together to actually get to the so called new identities as follows

$$\begin{aligned} \xi_{1i} (p^i \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p} - p_c \epsilon^{a_0 \dots a_{p-1} c} H_{a_0 \dots a_{p-1}}^i) &= 0 \\ \xi_{1i} k_{3c} k_{2a} (-p_d \epsilon^{a_0 \dots a_{p-3} cad} H_{a_0 \dots a_{p-3}}^i + p^i \epsilon^{a_0 \dots a_{p-2} ca} H_{a_0 \dots a_{p-2}}^i) &= 0 \end{aligned} \quad (65)$$

to the gauge field which means that if we replace  $\xi_{2a} \rightarrow k_{2a}$  the whole S-matrix vanishes where the following points are needed. In the first term of  $\mathcal{A}_2$  one has to apply momentum conservation along the world volume of brane  $-k_{3c} - k_{2c} = p_c + k_{1c}$  and apply the following identity

$$p_c \epsilon^{a_0 \dots a_{p-1} c} = 0.$$

We need to apply momentum conservation in  $\mathcal{A}_3$ 's first term, that is  $-k_{1d} - k_{2d} = p_d + k_{3d}$  and then draw particular attention to the fact that this part of the S-matrix involves  $k_{3d} k_{3c} \epsilon^{a_0 \dots a_{p-2} dc} C_{a_0 \dots a_{p-2}}^i$  which is zero because of the antisymmetric property of the  $\epsilon$  tensor. Likewise, we need to replace  $k_{3c} + k_{2c} = -p_c - k_{1c}$  and  $k_{1d} k_{1c} \epsilon^{a_0 \dots a_{p-2} dc} C_{a_0 \dots a_{p-2}}^i = 0$ . Finally we apply momentum conservation to the last term of (67) and note to the point that  $p_c \epsilon^{a_0 \dots a_{p-2} ca} C_{a_0 \dots a_{p-2}}^i = 0$  plays the crucial role in checking the Ward identity.

The last remark about the asymmetric picture of the S-matrices is that , one finds out all the entire contact interactions of string theory amplitudes, properly. As an instance this amplitude includes several contact interaction terms like the first term and the last terms of (67) which could be missed in its symmetric picture of (64).

## 11 Conclusion

We have derived scattering amplitudes of all three , four and five point BPS and non-BPS mixture of a closed string Ramond-Ramond , a scalar field , gauge and tachyons in their all different pictures of both world volume and transverse directions (for general  $p, n$  cases) of type IIA (IIB) String theory.

In particular we have shown that if we carry out the calculations of higher point functions in asymmetric picture of Ramond-Ramond (taking its vertex operator in terms of its potential  $C^{-2}$ ) and scalar field in zero picture then, various equations must be kept fixed for BPS branes, the entire contact interactions can be definitely obtained and most importantly the Ward identity associated to the gauge field is also satisfied.

More accurately, by direct calculations on upper half plane, we have also observed that some of the Bianchi identities (that must be true) for BPS branes can not be necessarily applied to non-BPS amplitudes, otherwise the whole S-matrix might be vanished. Indeed in the presence of the scalar field and RR, the terms carrying momentum of RR in transverse



directions  $(p^i, p^j)$  play important role in the entire form of the S-matrix and one has to keep them in five point functions.

We expect it to be true for higher point functions of string theory amplitudes and it would be nice to check it directly. It would also be important to deal with some other subtleties of the perturbative string theory [34].

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